CONSTRAINT CONTROL OF A SIMULATED ORE CRUSHING CIRCUIT

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by

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ABSTRACT

Crushing is an integral part of the process of recovering valuable minerals from an orebody, where the size of the ore is progressively reduced to be suitable for further processing. The goal of optimising a crushing circuit is to maximise the production of crushed ore. There are a number of advanced control techniques that may be suitable for optimising a crushing circuit, such as multi-loop PID control, Model Predictive Control and Fuzzy Logic Control.

The aim of this masters’ thesis was to compare PID and Fuzzy Logic Control strategies when applied to a crushing circuit to control conveyor power and bin level. Comparison was made by examining the effectiveness and the level of effort required to commission and tune each control strategy. This was performed by applying the control strategies to a dynamic model that was configured in the MATLAB/Simulink platform. Unit models were developed including a dynamic model of a belt conveyor that had the capability of dynamically determining the power draw.

In terms of productivity the PID strategy was slightly better than Fuzzy Logic Control, but there were wide variations in the manipulated variable which may not be desirable in some circumstances. The Fuzzy Logic Control strategy required more effort to configure and tune compared to the PID strategy. It was found that the configuration of the Fuzzy Logic Controller could affect performance and so the controller design as well as tuning should be reviewed during the tuning process. Performance of the Fuzzy Logic Controller was improved with an iterative tuning approach which suggests that further improvement may be possible with additional tuning effort.
CHAPTER 1. INTRODUCTION

1.1 Context

Crushing is the first mechanical stage of the comminution process where the particle size of the mined ore is progressively reduced in order to aid the recovery of valuable minerals. The typical arrangement for a mineral processing plant is to have the crushing circuit located between an upstream run of mine stockpile and a downstream crushed ore stockpile. The presence of the crushed ore stockpile creates the need to maximise the production rate from the crushing circuit.

Planned or unplanned downtime of the crushing circuit causes the level of the crushed ore stockpile to decrease which in turn causes the particle size of the material drawn from the stockpile to coarsen. Coarsening feed for the downstream grinding operation often requires the throughput to be decreased to maintain process control, which causes lost production for the valuable mineral. Hence there is a desire to recover the crushed ore stocks as quickly as possible to minimise the adverse impact.

The optimisation of a crushing circuit in terms of productivity is well suited to automated control using an advanced control strategy. Advanced control strategies such as Constraint Control, Fuzzy Logic Control and Model Predictive Control are often discussed in the study of process control [1-3], however it seems that no approach is recommended over another. Real world crushing circuits are often subject to operational constraints associated with the equipment, such as the maximum power draw of a conveyor, minimum and maximum bin levels, and the supply rate of the ore. Any control strategy used for optimisation must respect these
constraints and ideally would run the circuit at a production rate that is just below the most relevant constraint at the time.

1.2 Research Objective

The objective of this study is to compare two control strategies, namely PID Control and Fuzzy Logic Control when applied to the control of an ore crushing circuit that includes various operational constraints. Each strategy will be configured as a two input one output over-ride control strategy to control conveyor power and bin level by manipulating feeder speed. The crushing circuit will be simulated by a dynamic model that includes disturbances that will excite the control strategies. The goal is to understand the advantages and complexities of each strategy so that an informed decision may be made as to the best approach for controlling a crushing circuit.

1.3 Research Questions

The following questions will be answered in this thesis:

RQ1: Can a dynamic model of a crushing circuit be built that includes circuit constraints?

RQ2: What are the relative advantages of a PID Control and a Fuzzy Logic Control strategy when applied to the control of a crushing circuit?

RQ3: What is the relative level of effort required to bring a PID Control and a Fuzzy Logic Control strategy into operation?
1.4 Structure of the Thesis

This thesis is presented in four chapters:

Chapter 2 presents findings from a literature review. The aim of literature review is threefold: to gain an understanding of what are the important factors that apply to controlling a crushing circuit, to review earlier studies in order to understand how constraints have been considered in the dynamic simulations, and to determine what features should be included in a dynamic model.

Chapter 3 describes how the work will be conducted so that the research questions may be answered.

Chapter 4 presents the results obtained from the simulations and discusses the significant findings.

Chapter 5 presents the conclusions drawn from this work and provides some ideas for future work in this field.
CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

In a mineral processing plant the crushing circuit is the first stage of ore processing after the initial fragmentation of the rock using explosives, where the size of the rock fragments is progressively reduced to that required by the downstream grinding process.

Interest in dynamic simulation of crushing circuits appears to have grown over the last decade, as shown by several Masters [10, 14] and Doctoral [16, 42] studies being performed at Chalmers University of Technology, Sweden. In these and other studies crushing circuits have been simulated to design and test control strategies, and as a somewhat convenient way of examining how changes to equipment affect the circuit productivity.

The review of the published literature has three main aims:

- To understand crushing circuits in terms of function, control objectives and constraints, which collectively provide a sense of what is important for control.

- To examine earlier dynamic simulation studies to assess the breadth of coverage and to determine how constraints within the circuits have been considered in the simulations.

- To conduct a detailed review of the approach taken to develop a dynamic model including understanding how the features of real world process have been included in the model.
Each of the above aims corresponds to a section in the literature review with a final section providing a summary of the findings.

2.2 Crushing Circuits

2.2.1 Overview of Crushing Circuits

The purpose of a mineral processing plant is to prepare the mined ore so that the valuable minerals may be extracted [5]. Most valuable minerals exist as fine grains that are dispersed within the ore matrix which need to be unlocked or liberated before separation and subsequent recovery may be performed [5]. Liberation of the valuable minerals is achieved by the process of rock fragmentation or comminution. In this process the particle size is reduced until the final product consists of relatively clean particles of the valuable mineral and the unwanted material or gangue [5].

Comminution is typically performed by dry crushing followed by wet grinding [5]. The purpose of the crushing unit operation is to reduce the ore size to that which is suitable for the grinding process [5]. The mechanism of crushing is the relatively slow compression of particles between rigid surfaces [4, 5].

A crushing circuit typically involves 2 or 3 stage crushing [6]:

- Primary crushing reduces ore from a top size that may be as much as 1.5 m to a top size of 100-200 mm which is suitable for further crushing.

- Secondary and Tertiary crushers progressively reduce the size of the ore to that which is suitable for the wet grinding process. Final product top size is typically between 5 and 20 mm, but this depends on the requirements of the grinding process.
Primary crushers are typically large jaw or gyratory crushers whereas secondary and tertiary crushers are typically cone crushers [6]. In addition to the crushers, a crushing circuit typically includes the following equipment:

- Screens: used to remove fines from the primary crusher feed so that packing may be avoided, or to separate sized material from oversize which is returned for additional crushing.

- Surge bins: used to maintain a supply of material to the crushers.

- Feeders: withdraw material from the surge bins.

- Belt conveyors: transport ore between equipment in the crushing circuit.

- Final stockpile or bin: store the final product and to provide surge capacity for the grinding process.

Figure 1 and Figure 2 show examples of crushing circuit flowsheets.
2.2.2 Crushing Circuit Control Objectives and Constraints

At a high level a crushing circuit has four control objectives [8]:

- Ensure safe operation of the circuit.
- Provide equipment protection.
- Stabilise key process variables.
- Optimise the circuit.

Safe operation involves providing protection for personnel and preventing a condition developing that poses a risk to personnel, such as preventing overloading that could cause material to fall from height. Equipment protection aims to prevent equipment damage during operation by implementing precautions such as ensuring structural loads and motor rated power are not exceeded. Stabilisation aims to control key process variables to the desired setpoint, such as screen loading, crusher level and
conveyor feed rate. Optimisation is usually concerned with achieving the highest possible production rate [8, 9].

To achieve the above control objectives a two layer control strategy may be used, such as that described by Muller et al. [8]. The two layer control strategy consists of a basic control layer that implements interlocks, sequences and feedback control loops, and a supervisory control layer that performs the optimisation functions and provides setpoints to the basic control layer. On some crushing circuits the supervisory control layer is absent and the operator performs the optimisation function [9].

Control functions associated with ensuring safe operation and protecting equipment are elements that limit the performance of the crushing circuit and may be referred to as constraints. Additional control functions or constraints may be added to minimise the amount of intervention required to return the circuit to full production [9]. An example of this is stopping a feeder on high conveyor power instead of stopping a conveyor on high power [9]. It may not be possible to restart the conveyor in a loaded condition which may require material to be manually removed, where as it is likely that a feeder could be restarted with only minimal downtime.

A typical crushing circuit would include the following constraints [8, 9]:

- Storage bin level – high limit to avoid overflowing the bin and a low limit to avoid direct impact of ore onto the bin base and/or feeders.
- Storage bin discharge rate – avoid spillage, maximum capability of the feeder due to design, avoid exceeding the rated motor power.
• Conveyor belt capacity – maximum loading to avoid spillage, run within structural load limits and avoid exceeding the rated motor power.

• Screening capacity – avoid exceeding the rated motor power.

These constraints are implemented as interlocks and feedback control limits in the basic control system, such as [9]:

• Maximum throughput setpoint limit.

• Maximum feeder speed limits.

• High bin level interlock for the storage bin feeding system.

• Low bin level interlock for the storage bin discharge feeder.

• High motor power interlock for feeders, belt conveyors, crushers and screens.

Constraints within a crushing circuit become particularly important when there is a need to maximise production [9]. The experience of the author is that there is often a need to maximise production as the crushed product storage has become depleted due to planned or unplanned downtime on the crushing circuit. When the product storage level is lower than normal the size distribution from the reclaim feeders becomes coarse and the downstream grinding operation suffers [9].

Maximising production rate would involve running as close as possible to the limit of most significant constraint without exceeding the limit and causing equipment to stop due to an interlock [9]. If the crushing circuit under manual control, the operator must pay careful attention to each of these constraints to ensure that
additional production loss does not occur. However maximising production is a function that is well suited to automatic control, which may be performed within the supervisory control layer.

2.3 Dynamic Modelling of Crushing Circuits

Dynamic modelling of crushing circuits has been viewed as an important technique by several researchers in order to investigate alternative control strategies [10, 11] and to explore the effects of physical or operating parameter changes on the circuit [13]. The most common platform for dynamic modelling is MATLAB/Simulink, and in most cases an output of the modelling effort was a library of individual models [10, 13, 15].

A useful approach for model development was presented by Itävuo [12], where steady state non-linear models are coupled with linear dynamics to obtain a dynamic model. This approach was used to develop dynamic models of a cone crusher, screen, conveyor and feeder which were then combined for the purpose of simulating a single stage mobile crushing plant. Results were presented when the simulation was subject to a moisture disturbance and a change to the crusher Closed Side Setting (CSS).

Asbjornsson [13] recognised that traditional simulations of plant performance are steady state and explored the notion that actual plant performance deviates from predicted due to dynamic effects, both gradual and discrete. The study included the development of a dynamic model of a secondary crushing circuit that included a grizzly screen, crusher feed bin and feeders, 3 cone crushers, screen feed bin and feeders, two product screens and the associated belt conveyors. The model was used to explore the effects of changing the coarse crusher CSS and fine crusher throw on
the production capacity of the circuit by setting various feed rates to the circuit. The simulations revealed that the circuit either reached steady state at a given production rate or became overloaded, with feeders stopping on a high bin level then restarting after the level had recovered. A key output of the modelling work was the development of a custom library of models in MATLAB/Simulink that could be linked together to model a complex plant.

Johansson [10] continued the work of Asbjornsson [14] moving further downstream in the crushing plant to construct a dynamic model of the tertiary crushing circuit. The aim of the work was to construct a model of the tertiary crushing circuit that was accurate to within 10% of the live plant performance so that equipment changes and advanced control applications could be evaluated. The model for the circuit was used to compare the current circuit control strategy with an MPC approach when various product size screens were used. The simulations revealed that the circuit was stabilised more rapidly when MPC was used, which in turn allowed a higher production rate to be approached with confidence. A key output of the modelling work was the development of a dynamic model for the High Pressure Grinding Roller (HPGR) crusher to capture the dynamics associated with speed changes, pressure and feed size changes. However the size change through the HPGR was a fixed reduction, achieving model simplicity over a more involved population balance approach.

Lindstedt and Bolander [11] developed a dynamic model of a primary gyratory crusher circuit consisting of a truck dump hopper, apron feeder, crusher and discharge conveyor. The aim of the work was to investigate if the level within the crusher could be controlled automatically, thereby relieving operators of a demanding
task. The dynamic model was used to develop and evaluate single loop PID and Linear Quadratic Regulator (LQR) control strategies for the crusher level. An important contribution of this work was the construction of a model for the discharge rate from the crusher based on the height of the rotating mantle and the level of material within the crusher. As the aim of the modelling and simulation was to explore level control, breakage of the crusher feed was not included in the model.

Constraints within the crushing circuit were included in the dynamic models to a limited extent, usually in the form of feeder interlock logic based on bin levels [10, 13]. Although the model included feeder interlocks, Asbjörnsson did not seek to run at the limit of the constraint, rather the feed rate was set for a given simulation run [13]. In contrast to the approach of Asbjörnsson, Johansson set a maximum throughput constraint on a recirculating conveyor when evaluating alternative control strategies [10]. The approach taken by Johansson seems more closely aligned to plant operating practices where the aim is to maximise production from the circuit.

Size change between the crusher feed and discharge was specified using selection and breakage functions with parameters obtained from plant survey data [13] or in the case of a HPGR crusher, fixed breakage was assumed [10].

### 2.4 Dynamic Modelling Elements

#### 2.4.1 Model Development

A crushing circuit consists of a number of production units each performing specific functions that are connected together. Similarly a model of a crushing circuit consists of unit models that are connected together to obtain the overall model of the entire circuit [14, 15]. Each unit model may consist of a number of sub-models that
perform functions such as mass balance, material tracking and size reduction, that when combined with process dynamics describe how the unit input stream is transformed into the output stream [12, 14]. Unit models often output process data such as level and power which may be used as inputs to the modelled control system.

The above approach has the advantages of allowing unit models to be developed and tested in isolation, creating a model library that may be deployed in future modelling efforts and facilitating upgrading where a unit model or sub-model may be replaced with an enhanced version [10].

Sections 2.4.2 through 2.4.7 describe the features of the various unit models as reported in the technical literature.

2.4.2  Model Connection Data Structure

It is common to standardise the data connection between unit models. A standardised data connection allows the unit models to be connected together in any arrangement to construct an overall model of a crushing circuit. The output of one unit model becomes the input for the next model.

The data structure for dry crushing includes particle size distribution, mass flowrate and material properties [10, 14, 15]. The properties data could be a single term such as hardness [15] or could include several material properties such as density, moisture content and work index [14]. For wet processing the data structure is expanded to include water volumetric flowrate [15].
2.4.3 Storage Bin

The storage bin unit model is required to serve two purposes: to track material between the inlet and outlet, and to provide a material level signal. Asbjornsson [16] proposed two approaches for a storage bin model: vertical segments and horizontal layers. The vertical segment approach divides the bin into a number of vertical slices and material is transported between the segments based on the angle of repose. The horizontal layer approach divides the bin into a first in, first out queue of several layers with perfect mixing assumed in each layer. The vertical segment model appears to be significantly more complex but appears to be useful in situations where the bin has multiple feed points.

2.4.4 Feeder

A feeder is typically a variable speed device that is used to adjust the flowrate of material to that required by the downstream processing unit. There are three main types of feeders used in a crushing plant: vibrating feeders, apron feeders and belt feeders.

The discharge rate for vibrating feeders has been modelled as a first order process with dead time [14], as shown in equation (1).

\[ G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}e^{-\theta s} \]  

(1)

Apron feeders and belt feeders have the same operating principle and have been modelled as a linear function of speed with no dynamic effects [10, 11]. This approach is based on the assumption of a constant load on the feeder per unit length. Johansson [10] analysed the discharge rate of a belt feeder and found that the
response was linear through much of the speed range but the rate became saturated at high speeds. Based on this observation it was recommended that linear feeder models could be extended to include the non-linear saturation effects.

2.4.5 Belt Conveyor

Belt conveyors transport material between processing units leaving the particle size distribution and material properties unchanged. It follows that fixed speed belt conveyors may be modelled as a pure delay element [12, 15]. Additional modelling challenges appear if the conveyor speed is variable or if the conveyor may be stopped and started due to an interlock such as a level downstream. Asbjornsson [16] proposed a state space model to meet these challenges which was also used in subsequent work by others [10]. The model allows material to be tracked along the length of the conveyor and allows the conveyor to be stopped without deleting material from the simulation.

Belt conveyor energy models are described in standards and specifications from conveyor equipment manufacturers [17]. The models contain no dynamic elements and are used to size the conveyor drive motor using the conveyor dimensions, roller spacing and material flowrate.

Zhang and Xia developed an energy model for a conveyor based on ISO5048 that was used in energy optimisation studies [17]. The final model was of the form shown in equation (2), where $V$ is the belt speed (m/s), $T$ is the throughput (t/h) and $\theta$ are model coefficients. Although the model coefficients could be obtained from parameters in ISO5048 the authors recommended that these should be obtained from operating data.
\[ P_T = \frac{V^2 T}{3.6} + \theta_1 T^2 V + \theta_2 V + \frac{T^2}{V} + \theta_4 T \] (2)

One manufacturer [18] provided an equation for determining the power requirement of a belt conveyor given the length \( L \) (m), elevation change \( H \) (m), speed \( S \) (m/s), mass of moving parts \( Q \) (kg/m) and capacity \( C \) (t/h) as shown in equation (3). Equation coefficients are friction factors \( F_e \) and \( F_l \) for empty and loaded conditions respectively and an equivalent length \( t_f \) was used to account for the end of belt effects.

\[ P_T = \frac{F_e (L + t_f) 3.6 QS}{367} + \frac{F_l (L + t_f) C}{367} + \frac{CH}{367} \] (3)

2.4.6 Screen

There are two aspects to be considered when modelling a screen: the size classification and the dynamics between feed and discharge.

Sbarbaro and del Villar [15] described a static approach to modelling a screen. Mass flow of each size fraction in the screen underflow and overflow were determined from a classification matrix, which in turn is obtained from plant survey data. Considering the screen to be a static element is a simplifying assumption but this may be justified as the transportation delays due to conveyors and storage bins may be expected to be significantly longer.

King [19] presented the Karra model of a screen which enables the classification efficiency of a given size to be determined from the cut size or \( d_{50} \) of the feed.
Asbjornsson [16] presented the Soldinger Stafhammar velocity model which may be used to estimate the residence time on the screen from the screen length, screen angle, throw and frequency.

2.4.7 Crusher

When a crusher is modelled there are two aspects to consider: throughput and product size distribution.

Throughput of a jaw crusher may be determined from crusher dimensions, throw, CSS and operating speed [21, 22]. Average feed size and material properties have been reported to influence throughput [21]. CSS appears to be a key parameter for determining the throughput as crusher manufacturers often include a throughput range as a function of CSS in data sheets [23, 24].

There are two methods of determining the product size from a jaw crusher. One view is that the feed to a primary crusher such as a jaw crusher has a small proportion of feed with size less than the Open Side Setting (OSS), and so almost all of the material is crushed [20, 22]. Product size distribution is then independent of the feed size distribution and may be estimated from the OSS. This view appears to be supported by crusher manufacturers who often supply approximate product size distribution data that is a function of the crusher OSS or CSS [23-25]. An alternative and more involved approach is to utilise a Whiten crusher model where the product size distribution is calculated using classification and breakage functions [21, 26]. The classification function determines if a particle of a given size will be selected for breakage, and the breakage function specifies the fragment sizes. Classification is modelled as a function of the crusher OSS and CSS and the breakage function is determined from material testing [21].
Throughput of a cone crusher has been reported to be dependent on the level within the crushing chamber, becoming a maximum value when the level was higher than the choke feed level [13].

Modelled product size distribution from a cone crusher has either been set based on plant data [13, 27] or obtained from classification and breakage functions [20, 28, 29]. As reported by King [20], there has not been much work to determine the classification and breakage functions from first principles and so the approach is to obtain these from measured size distributions from the crusher of interest.

While methods exist to determine the throughput and size distributions from crushers, these models do not contain dynamic elements. As already discussed in Section 2.3, appropriate dynamic elements could be combined with the static functions to obtain a dynamic crusher model.

### 2.4.8 PID Control

The PID controller is the workhouse for process control, used in over 95 percent of control loops, with most loops of PI type with no derivative component [30]. While the standard PI or PID function may be adequate for many control problems, the control algorithm is often enhanced with an anti-windup scheme, and this approach should be taken in both simulated and real control applications.

In a control loop the final element or actuator often has limits [11, 31]. When the controller output reaches an actuator limit, the feedback loop is effectively broken as the controller is no longer able to alter the process value and so there will be a difference between setpoint and process value [31]. With a persistent difference between process value and setpoint, this error will continue to be acted on by the
integral action and the integral term may become very large [11, 31]. This is known as integrator wind-up. Poor control can occur as the error must change signs for a long period so that the integral term can return to within the range of the actuator [31]. Integrator wind-up can also occur when an over-ride control strategy is deployed, as the non-selected controller is disconnected from the final element [32].

An anti-windup scheme aims to address this shortcoming of the standard PID algorithm. A common anti-windup strategy is called back-calculation, where the integral term is recalculated using the difference between the limited output and the controller output [31]. In the case of over-ride control, the difference between the selected output and controller output is used in the recalculation [32, 33]. The back calculation anti-windup scheme is available within the MATLAB/Simulink PID function, as shown in Figure 3.

![Figure 3 MATLAB/Simulink PID Controller Anti-Windup](image)

**Figure 3 MATLAB/Simulink PID Controller Anti-Windup [34].**
2.5 Summary

The aim of comminution in a mineral processing plant is to liberate the valuable mineral grains from the surrounding rock matrix. A crushing circuit is the first stage of comminution where the particle size of the as mined rock is reduced to that required for the downstream wet grinding process.

On operating crushing circuits there is often a need to maximise production in order to restore the inventory of the crushed product storage and so optimisation of the crushing circuit in terms of production becomes an operating objective. Modelling and simulation of crushing circuits are frequently motivated by optimisation, with the aim being to develop and evaluate a particular control strategy, or to evaluate the effect of modifying equipment physical or operating aspects. The extent to which the production from a crushing circuit may be increased is limited by constraints, which when exceeded will cause equipment to shut down via an interlock. Constraints are usually implemented in the basic control layer to protect equipment from damage from excessive feeder speeds, high or low storage bin levels, high motor power and excessive conveyor loading.

Dynamic modelling of a crushing circuit involves developing individual unit models for equipment then connecting the unit models together. A standardised data structure allows the unit models to be connected in any arrangement to construct a complex overall model of a crushing circuit. The modelling approach may be relatively simple, including level and throughput only, or may be more complex including particle breakage and classification functions to model size distribution. Particle breakage and classification functions are obtained from measured size distributions on an operating plant.
While dynamic models for crushing circuit components have been well developed there are several areas where enhancements may be made:

- Including constraints such as conveyor motor power within the modelled control system.
- Including operational aspects in the simulation, such as material supply limitations and stoppages to clear metal from the crusher feed.
- Developing a dynamic model for jaw crusher product size and throughput.
- Simplifying the approach to modelling size distributions by only including a few discrete sizes that relate to screen apertures.
CHAPTER 3. METHODS

3.1 Modelling

3.1.1 Crushing Circuit Overview

The crushing circuit configuration selected for study is a primary-secondary crushing circuit that would typically be used to prepare feed for a wet grinding circuit [41]. A schematic diagram of the crushing circuit is shown in Figure 4.

Run of mine (ROM) ore is drawn from the ROM Bin and fed to the Primary Jaw Crusher via the Vibrating Grizzly Screen which removes the finer size fractions from the crusher feed. Ore is crushed in the Jaw Crusher followed by additional crushing of oversize material in the Secondary Cone Crusher. The Product Screen directs oversize material to the Coarse Ore Bin and sends correctly sized material to the stockpile. The Coarse Ore Bin provides surge capacity between processing stages. For the present study the ROM bin was treated as a material source and the stockpile as a material sink and as such these components had no influence on the performance of the simulation.

The crushing circuit under study is hypothetical, in that the physical dimensions, component ratings and other parameters are not based on a real world circuit. However this limitation is not viewed to be serious for the present study as, if required, parameter values could be adapted to match those of a given circuit. Appendix A contains the parameter values assigned to the various components of the crushing circuit.
3.1.2 Parameter Estimation using Mass Balance

As discussed in Section 2.4.7 modelling the size change through a crusher requires the classification and breakage functions to be estimated for each size fraction. With size distribution data available for the crusher feed and product a method similar to that performed by Ndhlala [29] would be followed:

- Obtain particle breakage data from test work or a model
- Define the equation used to model the crusher
- Solve the equation for missing terms: classification or selection function.

In the present study particle size distribution data from a real world operation is not available and so an alternative approach must be followed to obtain model parameters. The approach selected was to develop a steady state mass balance for the crushing circuit.

The mass balance was configured in Microsoft Excel and was solved iteratively due to the recirculation of product screen oversize via the Cone Crusher. Basis of the mass balance is described in Appendix C. The mass balance was used to obtain the following parameters:
• Nominal feed size distribution for the crushing circuit.

• Selection function for the Cone Crusher, which is the proportion of particles selected for breakage.

The nominal size distribution was that which produced 30% fines in the combined product from the Vibrating Grizzly Screen and the Jaw Crusher. Parameter values in the Selection function were varied to obtain a recirculating ratio for the Cone Crusher of 1.9. The targeted values of 30% fines and recirculating ratio of 1.9 were set based on the author’s experience observing several operating crushing circuits.

Results obtained in the mass balance for the nominal feed size and the Selection function are shown in Sections 3.1.3 and 3.1.6 respectively.

3.1.3 Particle Size

Particle Size Distribution will be based on ISO-3301 screen sizes. A simplified list of screen sizes was selected to provide a balance between size and complexity of the model and the ability to achieve sufficient resolution in the simulation. This simplification is thought to be valid as the simulation goals are concerned with comparing control strategies instead of predicting the size distribution of the final product. The selected sizes are shown in Table 1 along with the general behaviour of the size fractions.
Table 1 – Particle Sizes and General Behaviour.

<table>
<thead>
<tr>
<th>Size Designation</th>
<th>Size Range (mm)</th>
<th>Particle Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oversize (OS)</td>
<td>+160</td>
<td>Grizzly Screen Oversize</td>
</tr>
<tr>
<td>Coarse 1 (C1)</td>
<td>+115 -160</td>
<td></td>
</tr>
<tr>
<td>Coarse 2 (C2)</td>
<td>+80 -115</td>
<td>Grizzly Screen Oversize</td>
</tr>
<tr>
<td>Intermediate 1 (I1)</td>
<td>+56 -80</td>
<td>Product Screen Oversize</td>
</tr>
<tr>
<td>Intermediate 2 (I2)</td>
<td>+40 -56</td>
<td></td>
</tr>
<tr>
<td>Fines (F)</td>
<td>-40</td>
<td>Product Screen Undersize</td>
</tr>
</tbody>
</table>

Feed sizes were calculated using a Swebrec function [35] which determines the proportion of material less than a given size, as shown in equation (4):

\[ p(d_i) = \frac{1}{1 - g(d_i)} \]  

(4)

with

\[ g(d_i) = \left[ \ln(d_{max}/d_i)/\ln(d_{max}/d_{50}) \right]^b \]  

(5)

where:

- \( d_i \) is the particle size
- \( b \) is a curve calculation parameter.

Maximum particle size \( d_{max} \) was set to 400 mm and \( b \) was set to 3.0 to obtain all feed size distributions. To obtain the nominal feed size distribution the size for 50% passing \( d_{50} \) was varied so that the combined product from the Grizzly Feeder
and Jaw Crusher contained 30% Fines. To obtain size distributions for fine and coarse feed, the value of $d_{50}$ was varied by 20 mm relative to the nominal feed size distribution value. Table 2 shows the fine, nominal and coarse feed size distributions for the crushing circuit model.

**Table 2 – Crushing Circuit Feed Particle Size Distributions.**

<table>
<thead>
<tr>
<th>Size Designation</th>
<th>Mass Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fine Feed</td>
</tr>
<tr>
<td>OS</td>
<td>15.26</td>
</tr>
<tr>
<td>C1</td>
<td>15.93</td>
</tr>
<tr>
<td>C2</td>
<td>18.19</td>
</tr>
<tr>
<td>I1</td>
<td>14.63</td>
</tr>
<tr>
<td>I2</td>
<td>10.06</td>
</tr>
<tr>
<td>F</td>
<td>25.93</td>
</tr>
<tr>
<td>$d_{50}$ (mm)</td>
<td>80.0</td>
</tr>
</tbody>
</table>

### 3.1.4 Belt Conveyor

The Belt Conveyor Model must be capable of accounting for the transportation delay along the conveyor and must provide a dynamic estimate of the conveyor power.

The belt conveyor will be modelled as a pure delay element as indicated by Itävuo et al. [12] and also Sbarbaro and del Villar [15]. Material will be tracked along the conveyor by dividing the conveyor into multiple length segments and forming a first in first out queue, which performs an equivalent function to the state space approach described by Asbjørnsson [16].
A dynamic power calculation will be used to estimate the conveyor power as a function of feed rate, speed and conveyor dimensions. The dynamic calculation was derived from a static power calculation described in [18]. The derivation is shown in Appendix B.

Variables for the Belt Conveyor Model are shown in Table 3, and Figure 5 shows the functional structure of the dynamic model.

Table 3 – Belt Conveyor Model Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>Boolean</td>
<td>Input</td>
<td>Conveyor Running</td>
</tr>
<tr>
<td>F</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>D</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Material Vector</td>
</tr>
<tr>
<td>P</td>
<td>kW</td>
<td>Output</td>
<td>Belt Conveyor Power</td>
</tr>
<tr>
<td>L</td>
<td>m</td>
<td>Parameter</td>
<td>Length from feed point to discharge</td>
</tr>
<tr>
<td>H</td>
<td>m</td>
<td>Parameter</td>
<td>Elevation change from feed point to discharge</td>
</tr>
<tr>
<td>S</td>
<td>m/s</td>
<td>Parameter</td>
<td>Belt speed when running</td>
</tr>
<tr>
<td>l</td>
<td>m</td>
<td>Parameter</td>
<td>Conveyor segment length</td>
</tr>
<tr>
<td>n</td>
<td>Integer</td>
<td>Parameter</td>
<td>Number of segments</td>
</tr>
<tr>
<td>F_e</td>
<td>Parameter</td>
<td></td>
<td>Friction Factor, empty [18]</td>
</tr>
<tr>
<td>F_l</td>
<td>Parameter</td>
<td></td>
<td>Friction Factor, loaded [18]</td>
</tr>
<tr>
<td>T_f</td>
<td>m</td>
<td>Parameter</td>
<td>Terminal Friction [18]</td>
</tr>
<tr>
<td>Q</td>
<td>kg/m</td>
<td>Parameter</td>
<td>Mass of moving parts [18]</td>
</tr>
</tbody>
</table>
There are several functions within the Belt Conveyor Model:

- **Blend First Segment**: Integrate the feed mass flow rate to obtain the total segment mass (kg). Obtain average size distribution and hardness values for the segment.

- **Track belt position**: Determine when one segment length has passed the feed point. When a segment has passed send a trigger pulse to other functions.

- **Array Shift**: When the trigger pulse is on, shift the elements in Segment Array. Final element in the array is shifted to the Discharge function. Data from the Blend First Segment function is shifted to array position 1.

- **Discharge**: Convert the mass of the final array element into a mass flow rate. Form the Discharge material vector from the mass flow rate, size distribution and hardness.
• Power Model: Calculate the conveyor power from the total mass on the belt and the conveyor speed.

3.1.5 Jaw Crusher

The main function of the Jaw Crusher Model is to perform size reduction on the feed material to produce the product. To simplify the Jaw Crusher Model a constant delay between feed and discharge will be assumed. Following from this assumption, the model will not predict level within the crushing chamber. Avoiding a level estimation is consistent with jaw crusher observations by the author where the crusher usually runs almost empty, and the level only increases for a short time as large rocks are processed.

Product size distribution from the Jaw Crusher Model will be determined using the method described by King [20], as shown in equation (6):

\[
P(d_i) = 1 - \exp \left[-\left(\frac{r_i}{K_u}\right)^{1.5}\right]
\]  

(6)

with

\[
r_i = \frac{d_i}{OSS}
\]

(7)

\[
K_u = \left[ \ln \left( \frac{1}{1 - P_T} \right) \right]^{-0.67}
\]

(8)

where:

\( P(d_i) \) is the percent passing size \( d_i \)

\( d_i \) is the particle size
\( r_i \) is the size relative to the open side setting

OSS is the crusher open side setting

\( \rho_T \) is the material characteristic, the fraction of the product smaller than OSS

Table 4 shows the Jaw Crusher Model product size distribution calculated using an OSS of 120 mm and a material characteristic of 0.82 for hard tough materials with a crusher work index greater than 13 kWh [20]. The parameter value for OSS was based on a reasonable estimate by the author and this was necessary as no real world parameters were available for the current study.

<table>
<thead>
<tr>
<th>Size</th>
<th>Size (mm)</th>
<th>Passing (%)</th>
<th>Retained (%)</th>
<th>Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS</td>
<td>160</td>
<td>92.9</td>
<td>7.1</td>
<td>7.1</td>
</tr>
<tr>
<td>C1</td>
<td>115</td>
<td>80.1</td>
<td>12.8</td>
<td>12.8</td>
</tr>
<tr>
<td>C2</td>
<td>80</td>
<td>60.8</td>
<td>19.3</td>
<td>19.3</td>
</tr>
<tr>
<td>I1</td>
<td>56</td>
<td>42.2</td>
<td>18.6</td>
<td>18.6</td>
</tr>
<tr>
<td>I2</td>
<td>40</td>
<td>28.2</td>
<td>14.0</td>
<td>14.0</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td>28.2</td>
</tr>
</tbody>
</table>

Variables for the Jaw Crusher Model are shown in Table 5, and Figure 6 shows the functional structure of the dynamic model.
Table 5 – Jaw Crusher Model Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>D</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Material Vector</td>
</tr>
<tr>
<td>p</td>
<td>%</td>
<td>Parameter</td>
<td>Product size distribution from Table 4</td>
</tr>
<tr>
<td>T_d</td>
<td>s</td>
<td>Parameter</td>
<td>Crusher Dead Time</td>
</tr>
</tbody>
</table>

Figure 6 Jaw Crusher Model Functional Structure

3.1.6 Cone Crusher

As with the Jaw Crusher, the main function of the Cone Crusher Model is to perform size reduction on the feed material to produce the product. A second function is to estimate the level in the crushing chamber.

Size reduction was estimated by subjecting each particle size to a single stage breakage event: particles were selected for breakage and converted into fragments using a breakage function [5]. After the breakage event both the unbroken particles and broken fragments were discharged from the crusher model. This process is
described by equation (9), where the first term is the mass flow rate of fragments of size $i$ and the second term is the mass flow rate of unbroken particles of size $i$:

$$p_i = \sum_{j=i+1}^{n} B(i, j)S_j f_j + (1 - S_i)f_i$$  \hspace{1cm} (9)$$

where $S_j$ is the particle selection function and $B(i, j)$ is the breakage function. Values for $S_j$ were estimated using the steady state mass balance as described in Section 3.1.2 and are shown in Table 6. The breakage function, shown in equation (10) was obtained from King [20]:

$$B(d_1, d_2) = K \left(\frac{d_2}{d_1}\right)^{n_1} - (1 - K) \left(\frac{d_2}{d_1}\right)^{n_2}$$ \hspace{1cm} (10)$$

where $B(d_1, d_2)$ is the fraction of particle fragments smaller than $d_1$ obtained from breakage of a single particle of size $d_2$. Constants $K$, $n_1$ and $n_2$ are material parameters and were assigned values of 0.3, 0.45 and 3.2 respectively from King [20] allowing the breakage matrix to be populated, as shown in Table 7.

<table>
<thead>
<tr>
<th>Table 6 – Cone Crusher Selection Function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Selection</td>
</tr>
</tbody>
</table>
Table 7 – Cone Crusher Breakage Matrix.

<table>
<thead>
<tr>
<th>Fragment Size $d_1$ (mm)</th>
<th>Original Particle Size $d_2$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td>160</td>
<td>23.6</td>
</tr>
<tr>
<td>115</td>
<td>18.4</td>
</tr>
<tr>
<td>80</td>
<td>14.9</td>
</tr>
<tr>
<td>56</td>
<td>12.5</td>
</tr>
<tr>
<td>40</td>
<td>30.5</td>
</tr>
<tr>
<td>Total (%)</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Level within the crushing chamber will be estimated by integrating the difference between feed and discharge for each size fraction. Total discharge rate from the crusher will be based on the work by Asbjornsson et al. [13]. Assuming a constant bulk density of the material allows the result of the integration to be converted to a percent level using the crushing chamber capacity.

Asbjornsson et al. [13] reported that throughput of the crusher was a function of the level in the crushing chamber, reaching a maximum rate when the crusher is choke fed. The relationship between throughput and level is shown in equation (11):

$$R_L = R_{\text{max}}[1 - \exp(KL)]$$

(11)

where $R_L$ is the capacity as a function of level $L$ and $R_{\text{max}}$ is the maximum capacity. The value of parameter $K$ may be found to be 7.675, making the approximation that 99% of the maximum capacity is reached at a level of 60%.
The final element of the Cone Crusher Model is the transportation delay between the crusher feed and crusher discharge. To avoid the complexity of embedding a bin level model within the Cone Crusher Model, well mixed conditions within the crushing chamber are assumed.

Variables for the Cone Crusher Model are shown in Table 8, and Figure 7 shows the functional structure of the model.

Table 8 – Cone Crusher Model Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>D</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Material Vector</td>
</tr>
<tr>
<td>L</td>
<td>%</td>
<td>Output</td>
<td>Crushing Chamber Level</td>
</tr>
<tr>
<td>C</td>
<td>t</td>
<td>Parameter</td>
<td>Crushing Chamber Capacity at 100% level</td>
</tr>
<tr>
<td>L_{choke}</td>
<td>%</td>
<td>Parameter</td>
<td>Chamber Level for 100% throughput</td>
</tr>
<tr>
<td>R_{max}</td>
<td>t/h</td>
<td>Parameter</td>
<td>Maximum throughput rate</td>
</tr>
<tr>
<td>K</td>
<td>1/%</td>
<td>Parameter</td>
<td>Throughput calculation parameter</td>
</tr>
<tr>
<td>S</td>
<td>vector</td>
<td>Parameter</td>
<td>Selection for breakage vector</td>
</tr>
<tr>
<td>B</td>
<td>matrix</td>
<td>Parameter</td>
<td>Breakage matrix (5x5)</td>
</tr>
</tbody>
</table>
There are several functions within the Cone Crusher Model:

- **Integrator**: For each size fraction integrate the difference between the crusher feed and the breakage feed. Determine the total mass and convert to a percent of the crushing chamber capacity.

- **Blending**: Determine the average size distribution within the crushing chamber.

- **Crushing Rate**: Determine the total rate of material drawn from the crushing chamber for breakage.

- **Breakage Feed**: Obtain the feed rate of each size fraction to the Breakage function by combining the average size distribution with the total rate of material.

- **Breakage**: Convert the Breakage Feed to the Crusher Discharge using the Selection Vector $S$ and the Breakage Matrix $B$. 

\[ \text{Figure 7 Cone Crusher Model Functional Structure} \]
3.1.7 Bin Model

The Bin Model will be based on the layered model described by Asbjornsson [16]. The Bin Model is used to apply a variable delay between the bin feed and discharge that is dependent on the level in the bin. The Bin Model will output a level in percent that may be used for other control functions in the simulation. A constant bulk density of the material was assumed in order to simplify the conversion of total mass to percent level.

Variables for the Bin Model are shown in Table 9, and Figure 8 shows the functional structure of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>t/h</td>
<td>Input</td>
<td>Material flow rate drawn by the feeder</td>
</tr>
<tr>
<td>F</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>D</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Material Vector</td>
</tr>
<tr>
<td>L</td>
<td>%</td>
<td>Output</td>
<td>Bin Level</td>
</tr>
<tr>
<td>C</td>
<td>t</td>
<td>Parameter</td>
<td>Bin Capacity at 100% level</td>
</tr>
<tr>
<td>n</td>
<td>Integer</td>
<td>Parameter</td>
<td>Number of layers</td>
</tr>
<tr>
<td>k</td>
<td>Integer</td>
<td>Parameter</td>
<td>Number of layer being filled</td>
</tr>
<tr>
<td>L</td>
<td>vector</td>
<td>Parameter</td>
<td>Layer Material Vector</td>
</tr>
</tbody>
</table>
There are several functions within the Bin Model:

- **Blend Filling Segment**: Integrate the feed mass flow rate to obtain the total segment mass (kg). Obtain average size distribution and hardness values for the segment. Increase filling layer k by 1 when the layer is filled.

- **Discharge**: Remove mass from array element 1 at the rate specified by R. Form the Discharge material vector from the mass flow rate, size distribution and hardness. Call the Array Shift function when layer 1 is empty.
- Array Shift: Shift contents of all array elements down by 1 and decrease filling layer \( k \) by 1.

- Level: Calculate the percent filled from the total mass in all layers.

3.1.8 Feeder

The Feeder Model will be based on that described by Asbjornsson [14]. The feeder is modelled as a First Order Plus Dead Time (FOPDT) process including discharge rate saturation as a non-linear element. In the model the first order lag accounts for the effect of speed on the discharge rate from the feeder, and the dead time element accounts for the transportation delay between feeder inlet and outlet. Variables for the Feeder Model are shown in Table 10, and Figure 9 shows the functional structure of the dynamic model.

### Table 10 – Feeder Model Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>%</td>
<td>Input</td>
<td>Feeder Speed, 0-100%</td>
</tr>
<tr>
<td>Run</td>
<td>Boolean</td>
<td></td>
<td>Feeder Running</td>
</tr>
<tr>
<td>F</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>D</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Material Vector</td>
</tr>
<tr>
<td>R</td>
<td>t/h</td>
<td>Output</td>
<td>Material flow rate from feeder</td>
</tr>
<tr>
<td>K</td>
<td>t/h/%</td>
<td>Parameter</td>
<td>Feeder Gain</td>
</tr>
<tr>
<td>T</td>
<td>s</td>
<td>Parameter</td>
<td>Feeder First Order Time Constant</td>
</tr>
<tr>
<td>( T_d )</td>
<td>s</td>
<td>Parameter</td>
<td>Feeder Dead Time</td>
</tr>
<tr>
<td>( R_{\text{max}} )</td>
<td>t/h</td>
<td>Parameter</td>
<td>Maximum rate from feeder</td>
</tr>
</tbody>
</table>
3.1.9 Screen

The Screen Model will be based on that described by Sbarbaro and del Villar [15], which is a mass balance between the feed, oversize and undersize for each particle size. Equations (12) and (13) describe the mass balance for the screen model:

\[ p_{o,i} = C_i f_i \]  
\[ p_{u,i} = (1 - C_i) f_i \]

where:

f is the screen feed mass flow rate

\( p_0 \) is the mass flow rate of the oversize

\( p_u \) is the mass flow rate of the undersize

C is the classification function
The model of Sbabaro and del Villar will be enhanced to account for the transportation time across the screen deck in the form of a delay between feed and discharge for the oversize material. Variables for the Screen Model are shown in Table 11, and Figure 10 shows the functional structure of the dynamic model.

### Table 11 – Screen Model Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Usage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>vector</td>
<td>Input</td>
<td>Feed Material Vector</td>
</tr>
<tr>
<td>( D_o )</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Vector - Oversize</td>
</tr>
<tr>
<td>( D_u )</td>
<td>vector</td>
<td>Output</td>
<td>Discharge Vector - Undersize</td>
</tr>
<tr>
<td>( C )</td>
<td>vector</td>
<td>Parameter</td>
<td>Screen Classification vector</td>
</tr>
<tr>
<td>( T_d )</td>
<td>s</td>
<td>Parameter</td>
<td>Layer Material Vector</td>
</tr>
</tbody>
</table>

![Figure 10 Screen Model Functional Structure](image)
3.2 Process Control

3.2.1 Equipment Protection Functions

As discussed in Section 2.2.2 a crushing circuit usually has equipment protection functions implemented in the basic control layer, such as interlocks based on bin levels and conveyor belt power. The protection functions become constraints for the crushing circuit since if any interlock is activated there is a production impact to some extent. Table 12 shows the equipment protection functions that will be implemented in the simulation. Each function will be implemented with hysteresis so that protection functions do not activate and immediately recover which could lead to instability in the simulation.

Table 12 – Equipment Protection Functions.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Interlock</th>
<th>Action</th>
<th>Restart Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV01</td>
<td>Power &gt; High</td>
<td>Stop FE01</td>
<td>Power &lt; High</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BN02</td>
<td>Level &gt; High</td>
<td>Stop FE01</td>
<td>Level &lt; High</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BN02</td>
<td>Level &lt; Low</td>
<td>Stop FE02</td>
<td>Level &gt; Low</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR02</td>
<td>Level &gt; High</td>
<td>Stop FE02</td>
<td>Level &lt; High</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2.2 Stabilisation Functions

Stabilisation functions are present in a control system to stabilise key process variables so that optimisation functions have an effective operating base, as discussed in Section 2.2.2. In the crushing circuit simulation, the level in the Cone Crusher CR02 will be controlled by varying the speed of feeder FE02. The purpose of the
level control is twofold: avoid activating CR02 high level interlock to promote stability, and maximising throughput of crusher CR02 by maintaining the level greater than the level for choke feeding. Level control will be implemented by a single PI controller.

3.2.3 Optimisation Functions

As discussed in Section 2.2.2 the purpose of the optimisation functions is to maximise the production from the crushing circuit, which in an ideal situation would involve running close but not exceeding constraints within the circuit.

Implementing the optimisation functions will allow research questions RQ2 and RQ3 to be addressed, which were comparing the effectiveness of advanced control strategies and comparing the level of effort to implement the control strategies respectively. The optimisation functions will take the form of two control strategies, PID Control and Fuzzy Logic Control, that will control the feed rate to the circuit subject to Coarse Ore Bin BN02 level and belt conveyor CV01 power constraints. Each of these strategies will be configured as a two input one output control configuration with the final feeder speed output obtained via a low select function.

The PID Control strategy will utilise one PID controller for each of the bin level and conveyor power constraints. The PID controllers were tuned using the Lambda tuning approach [37]. Appendix D presents how the PID controllers were tuned.

The Fuzzy Logic Control strategy will utilise a 3x3 rule set for each of the bin level and conveyor power constraints. The 3x3 rule set will be formed from the fuzzy sets positive, zero and negative applied to the error relative to setpoint, and to the
error change. Table 13 shows the 3x3 rule set with the controller output action assigned to each rule.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Error</th>
<th>Error Change</th>
<th>Output Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Positive</td>
<td>Positive</td>
<td>Negative Large</td>
</tr>
<tr>
<td>B</td>
<td>Zero</td>
<td>Positive</td>
<td>Negative Small</td>
</tr>
<tr>
<td>C</td>
<td>Negative</td>
<td>Positive</td>
<td>Zero</td>
</tr>
<tr>
<td>D</td>
<td>Positive</td>
<td>Zero</td>
<td>Negative Small</td>
</tr>
<tr>
<td>E</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>F</td>
<td>Negative</td>
<td>Zero</td>
<td>Positive Small</td>
</tr>
<tr>
<td>G</td>
<td>Positive</td>
<td>Negative</td>
<td>Zero</td>
</tr>
<tr>
<td>H</td>
<td>Zero</td>
<td>Negative</td>
<td>Positive Small</td>
</tr>
<tr>
<td>I</td>
<td>Negative</td>
<td>Negative</td>
<td>Positive Large</td>
</tr>
</tbody>
</table>

The output change from each rule is of the form shown in equation (14), where $z$ is the output change, $x$ is the error, $y$ is the error change and $a$, $b$ and $c$ are tuning parameters. Table 14 and Table 15 list the output changes for the Conveyor Power and Bin Level Fuzzy Controllers respectively with parameter values shown in Appendix D. Appendix D also presents the configuration of the fuzzy logic controllers and describes how the tuning parameters were calculated.

$$z = ax + by + c$$  \hspace{1cm} (14)
Sugeno aggregation [36] will be applied to the outputs from each 3x3 rule set to obtain the final output from the fuzzy controller. This aggregation technique uses the rule strength as the weighing factor to obtain a weighted average of the rule outputs [36].

**Table 14 – Conveyor Power Fuzzy Logic Controller Tuning Parameters.**

<table>
<thead>
<tr>
<th>Output Change</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Large</td>
<td>0</td>
<td>-2B</td>
<td>-C</td>
</tr>
<tr>
<td>Negative Small</td>
<td>0</td>
<td>-B</td>
<td>-C</td>
</tr>
<tr>
<td>Zero</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Positive Small</td>
<td>0</td>
<td>0.5B</td>
<td>0.5C</td>
</tr>
<tr>
<td>Positive Large</td>
<td>0</td>
<td>2B</td>
<td>C</td>
</tr>
</tbody>
</table>

**Table 15 – Bin Level Fuzzy Logic Controller Tuning Parameters.**

<table>
<thead>
<tr>
<th>Output Change</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Large</td>
<td>0</td>
<td>-2B</td>
<td>-C</td>
</tr>
<tr>
<td>Negative Small</td>
<td>0</td>
<td>-B</td>
<td>-C</td>
</tr>
<tr>
<td>Zero</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Positive Small</td>
<td>0</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Positive Large</td>
<td>0</td>
<td>2B</td>
<td>C</td>
</tr>
</tbody>
</table>
3.3 Simulations

3.3.1 Simulation Cases

Various simulations of the crushing circuit were performed to explore the performance of the two control strategies while subject to two separate disturbances: changes in feed size and metal detections on the Crusher Feed Conveyor CV02. For the PID control strategy, simulations for 3 different tunings of the bin level controller were performed, while tuning of the belt conveyor power controller was kept constant. For the Fuzzy Logic Control Strategy an initial simulation case was performed followed by additional cases after adjustments were made to the fuzzy logic controllers. For both the PID and Fuzzy Logic control strategies starting speed for Feeder FE01 was set to 40% to bring feed on at a reasonable rate and avoid an immediate overload on belt conveyor CV01.
Table 16 summarises the simulation cases that were performed and Appendix D includes full details of the controller tuning and configuration that was used.
Table 16 – List of Simulation Cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Control</th>
<th>Disturbance</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>PID</td>
<td>Feed Size</td>
<td>$\lambda = 15$ min</td>
</tr>
<tr>
<td>Case 1a</td>
<td>PID</td>
<td>Feed Size</td>
<td>$\lambda = 10$ min</td>
</tr>
<tr>
<td>Case 1b</td>
<td>PID</td>
<td>Feed Size</td>
<td>$\lambda = 20$ min</td>
</tr>
<tr>
<td>Case 2</td>
<td>FLC</td>
<td>Feed Size</td>
<td>Initial Tuning</td>
</tr>
<tr>
<td>Case 2a</td>
<td>FLC</td>
<td>Feed Size</td>
<td>Tuning Iteration 1</td>
</tr>
<tr>
<td>Case 2b</td>
<td>FLC</td>
<td>Feed Size</td>
<td>Tuning Iteration 2</td>
</tr>
<tr>
<td>Case 3</td>
<td>PID</td>
<td>Metal Detection</td>
<td>$\lambda = 15$ min</td>
</tr>
<tr>
<td>Case 3a</td>
<td>PID</td>
<td>Metal Detection</td>
<td>$\lambda = 10$ min</td>
</tr>
<tr>
<td>Case 3b</td>
<td>PID</td>
<td>Metal Detection</td>
<td>$\lambda = 20$ min</td>
</tr>
<tr>
<td>Case 4</td>
<td>FLC</td>
<td>Metal Detection</td>
<td>Initial Tuning</td>
</tr>
<tr>
<td>Case 4a</td>
<td>FLC</td>
<td>Metal Detection</td>
<td>Tuning Iteration 1</td>
</tr>
<tr>
<td>Case 4b</td>
<td>FLC</td>
<td>Metal Detection</td>
<td>Tuning Iteration 2</td>
</tr>
<tr>
<td>Case 4c</td>
<td>FLC</td>
<td>Metal Detection</td>
<td>Membership Function</td>
</tr>
</tbody>
</table>

3.3.2 Disturbances

Two disturbances were applied to the simulation cases to elicit a response from the control strategies: changes in feed size and metal detections on the Crusher Feed Conveyor CV02.

Feed size changes were configured to occur on 2 hourly intervals in order to have sufficient time for the control response to reach steady state. The sequence of feed size disturbances is shown in Table 17.
Table 17 – Feed Size Disturbances.

<table>
<thead>
<tr>
<th>Simulation Start Time</th>
<th>Simulation End Time</th>
<th>Feed Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 h</td>
<td>2 h</td>
<td>Nominal</td>
</tr>
<tr>
<td>2 h</td>
<td>4 h</td>
<td>Fine</td>
</tr>
<tr>
<td>4 h</td>
<td>6 h</td>
<td>Nominal</td>
</tr>
<tr>
<td>6 h</td>
<td>8 h</td>
<td>Coarse</td>
</tr>
<tr>
<td>8 h</td>
<td>10 h</td>
<td>Nominal</td>
</tr>
<tr>
<td>10 h</td>
<td>12 h</td>
<td>Fine</td>
</tr>
</tbody>
</table>

Metal detection disturbances were modelled as a Poisson process, which is a process where events occur at random moments but at a constant average rate [39, 40].

The cumulative distribution function for a Poisson process is shown in equation (15):

\[ P(x) = 1 - e^{-\lambda x} \]  

where:

\( x \) is elapsed time

\( \lambda \) is (average time between events)\(^{-1}\)

The cumulative distribution function was used to obtain the time between metal detection events by assigning a random value on the interval (0,1) to \( P(x) \) and solving equation (15) for \( x \) with \( \lambda \) known [39, 40].
Metal detections were assumed to occur at an average rate of 30 per 12 hour shift, which gives an average time between events of 24 minutes. When a metal detection event was active, feeder FE02 and conveyor CV02 were stopped for 3 minutes. A sequence of random numbers was generated so that there were metal detection events across the full 12 hour simulation run. Event start and stop times were set as constants in the model so to be consistent between simulation cases.
CHAPTER 4. RESULTS AND DISCUSSION

4.1 Dynamic Model of Belt Conveyor Power

4.1.1 Simulation Results for Belt Conveyor CV01

Figure 11 shows simulation results for belt conveyor power and feedrate obtained from the first 2 hours for simulation Case 3. The figure shows power varying with both feed rate and feeder speed. The large changes in power and feed rate are caused by the on/off operation of feed to the secondary crusher due to metal detection events. Although power can be observed to vary with feed rate it is unclear from the figure if the relationship is static or dynamic.

The dynamic nature of the belt conveyor power model is revealed by examining how power varies with feedrate. Figure 12 shows a plot of conveyor power against feed rate for the first 30 minutes of Case 3 simulation. By following the curve in Figure 12 from the origin it can be seen that feedrate increases with power relatively constant followed by a large change in power. A similar effect is produced as feedrate decreases: feed rate changes first followed by a large decrease in power.

In the simulation CV01 feedrate is determined at the point where the product from the secondary crusher is combined with the product from the feeder and primary crusher. A change in the mass flowrate at this point produces a minor effect in power but the effect on power continues to develop as the material is transported along the conveyor to the discharge end.
Hence we may conclude that the belt conveyor model is dynamic in nature and appears to show the correct effects due to changes in conveyor feedrate.

**Figure 11 CV01 Power Trend, PID Control $\lambda=15$ minutes.**

**Figure 12 CV01 Power trajectory with Feedrate.**

### 4.1.2 Step Test Results for Belt Conveyor CV01

To determine the process reaction curve for CV01 power a step change in feeder FE01 speed was made. The process reaction curve was used to obtain an approximate FOPDT model for conveyor power using the method described by Wade [38]. The process reaction curve is shown in Figure 13 and Table 18 shows the characterisation parameters for CV01 power. The dimensionless process gain was
obtained using 180 kW as the full scale value for CV01 power. Results from the step test were used as a basis for tuning the conveyor power PID and Fuzzy Logic controllers.

Table 18 – Power Response Characterisation for CV01.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process Gain</td>
<td>kW / %</td>
<td>0.929</td>
</tr>
<tr>
<td>Process Gain (-)</td>
<td>(-)</td>
<td>0.516</td>
</tr>
<tr>
<td>Time Constant</td>
<td>s</td>
<td>22.0</td>
</tr>
<tr>
<td>Dead Time</td>
<td>s</td>
<td>16.8</td>
</tr>
</tbody>
</table>

Figure 13 Process Reaction Curve for CV01 Power.

4.1.3 Belt Conveyor Power Model Validation

The derivation for dynamic conveyor power shown in Appendix B showed that dynamic power may be calculated with time if the mass of material on the
conveyor can be determined. Construction of the Belt Conveyor Model with the conveyor length divided into multiple segments allowed the total mass to be determined by performing a summation on the array elements.

The final part in developing any model should be a validation step so that any results obtained due to the application of the model can be analysed with confidence. The validation step for the Belt Conveyor Power Model was not performed due to the difficulty in obtaining real world conveyor design data and operating parameters, and such there may be some doubt as to the validity of the results. However given an appropriate information set consisting of conveyor design data, throughput and power draw, validating the model is not expected to present any difficulties, and hence it is expected that the Belt Conveyor Model could be used to simulate real world conveyors if required.

4.2 Control Strategy Comparison – Size Change Disturbances

4.2.1 Overview

The overall results when size change disturbances were applied to the simulation are shown in Table 19 and Table 20. Table 19 presents the production obtained for the simulations as a total for the 12 hour simulation and normalised where 100% represents the highest 12 hour production value. Table 20 shows other data from the simulations:

- The number of times feeder FE01 stopped due to an interlock condition.
- The maximum speed for feeder FE01 in the first hour of the simulation.
- The largest speed change over a 5 minute period for feeder FE01 during hours 4 to 10 of the simulation.
The highest production was obtained from Case 1a which was the PID based control strategy with the highest gain tuning for BN02 level controller. This case was also successful in optimising the circuit during the initial run-up period with the feeder speed reaching the maximum of 80% during the first hour.

Table 19 – Production Results for Size Change Disturbances.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Control</th>
<th>Production (t)</th>
<th>Normalised (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>PID</td>
<td>5937</td>
<td>99.3</td>
</tr>
<tr>
<td>Case 1a</td>
<td>PID</td>
<td>5977</td>
<td>100.0</td>
</tr>
<tr>
<td>Case 1b</td>
<td>PID</td>
<td>5872</td>
<td>98.2</td>
</tr>
<tr>
<td>Case 2</td>
<td>Fuzzy Logic</td>
<td>5884</td>
<td>98.4</td>
</tr>
<tr>
<td>Case 2a</td>
<td>Fuzzy Logic</td>
<td>5890</td>
<td>98.5</td>
</tr>
<tr>
<td>Case 2b</td>
<td>Fuzzy Logic</td>
<td>5930</td>
<td>99.2</td>
</tr>
</tbody>
</table>

Table 20 – Additional Observations for Size Change Disturbances.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Feeder Stoppages</th>
<th>Feeder max speed 1st hour (%)</th>
<th>Feeder 5 min. Speed Changes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0</td>
<td>75.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Case 1a</td>
<td>0</td>
<td>80.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Case 1b</td>
<td>0</td>
<td>62.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>45</td>
<td>0.9</td>
</tr>
<tr>
<td>Case 2a</td>
<td>0</td>
<td>44.8</td>
<td>1.2</td>
</tr>
<tr>
<td>Case 2b</td>
<td>1</td>
<td>79.9</td>
<td>3.9</td>
</tr>
</tbody>
</table>
4.2.2 PID Control Performance

Figure 14 shows a 2 hour trend of CV01 conveyor power and feeder FE01 speed for the PID control strategy simulations subject to feed size disturbances. Tuning of CV01 power controller was found to be adequate for Case 1 as CV01 ran at the power setpoint until the feeder speed was decreased due to the action of BN02 level controller. As the power controller tuning was satisfactory the tuning was not adjusted for the subsequent PID control strategy based simulations subject to feed size or metal detection disturbances.

**Figure 14 CV01 Power Control under PID Control, feed size disturbances.**

Figure 15 shows a 12 hour trend of BN02 level and feeder FE01 speed for the PID control strategy simulations subject to feed size disturbances. All simulations were similar in that the feeder speed decreased as the bin level reached setpoint.
After the level setpoint was reached the feed size disturbances every 2 hours caused the feeder speed to adjust and the bin level was maintained close to the setpoint.

**Figure 15 BN02 Level Control under PID Control, feed size disturbances.**

4.2.3 *Fuzzy Logic Control Performance*

Figure 16 presents a 2 hour trend of CV01 conveyor power and feeder FE01 speed for the Fuzzy Logic control strategy simulations subject to feed size disturbances. From this figure it is apparent that conveyor power was not fully optimised during the initial stages of the simulation by any of the Fuzzy Logic control cases as the feeder did not reach the maximum value of 80%. Simulation Case 2b was the only simulation that successfully optimised production during the first 2 hours by controlling conveyor power at setpoint. However Case 2b also caused CV01 power to exceed the high power interlock value on one occasion, as shown by the
sudden decrease in power and feeder speed between 15 and 30 minutes simulation time.

![Figure 16 CV01 Power Control under Fuzzy Logic Control, feed size disturbances.](image)

Figure 16 shows a 12 hour trend of BN02 level and feeder FE01 speed for the Fuzzy Logic control strategy simulations subject to feed size disturbances. Cases 2 and 2a took approximately 6.7 and 7.5 hours respectively for BN02 level to reach the setpoint. After the level reached setpoint, level was controlled close to but not exactly at setpoint, performance that is adequate for a surge bin. Case 2b was much quicker to reach the level setpoint requiring 1.2 hours, but required another 3 hours to recover from a high level condition.
The motivation for changing the tuning of the Fuzzy Logic Controller is shown in Figure 17. Between simulation time 2 and 6 hours Case 2 showed a feeder speed that appeared to be unchanging although the level is well below the setpoint. With conveyor power and bin level below setpoint the circuit is not optimised and so the tuning was modified.

First attempt at addressing this was to modify the tuning for Rule I (negative error, negative change) for simulation Case 2a. The tuning change was applied to both the bin level and conveyor power controllers. The response was still unsatisfactory with feeder speed remaining steady as with Case 2.

A second attempt was made to optimise the simulation by applying the modified Rule I tuning to Rule F (negative error, zero change) for simulation Case 2b.
Again the tuning change was applied to both the bin level and conveyor power controllers. An improved result was obtained for Case 2b, with a higher feeder speed in the first hour, conveyor power running at setpoint and improved production over the 12 hour simulation.

4.2.4 Discussion

PID Control was found to be effective for both conveyor power control and bin level control with both parameters controlled to the respective setpoints. Conveyor power control required no tuning adjustments as the controller responded adequately to the power increase as the secondary crusher came on-line by reducing the feeder speed and avoiding a feeder trip due to exceeding the high power interlock. The highest production rate was achieved for the most aggressively tuned level controller which corresponded to the shortest arrest time for a disturbance.

Fuzzy Logic Control was found to achieve less production from the crushing circuit over a 12 hour period. The initial response of the Fuzzy Logic Controller was sluggish and the circuit was not optimised. With tuning adjustments made to Rule I (negative error, negative change) and Rule F (negative error, zero change), the Fuzzy Logic Controllers were shown to be capable of controlling both conveyor power and bin level. Tuning adjustments were made by an iterative process of running the simulation, reviewing the performance and modifying tuning parameters to address performance shortcomings.

From the feed size disturbance simulations, the expectation for the simulations with the more aggressive metal detection disturbances is that high production will be favoured by high gain PID tuning for the bin level controller, and that some tuning iterations may be required to optimise the Fuzzy Logic Controllers.
4.3 Control Strategy Comparison – Metal Detection Disturbances

4.3.1 Overview

The overall results when metal detection disturbances were applied to the simulation are shown in Table 21 and Table 22. As with the feed size disturbance simulations, production data is presented as a total for the 12 hour simulation and in normalised form.

The simulation case with the highest production was the same as that for the feed size disturbance simulations: PID based control strategy with the highest gain tuning for the bin level controller. This was anticipated from the results of the feed size disturbance simulations.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Control</th>
<th>Production (t)</th>
<th>Normalised (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 3</td>
<td>PID</td>
<td>4893</td>
<td>99.1</td>
</tr>
<tr>
<td>Case 3a</td>
<td>PID</td>
<td>4936</td>
<td>100.0</td>
</tr>
<tr>
<td>Case 3b</td>
<td>PID</td>
<td>4833</td>
<td>97.9</td>
</tr>
<tr>
<td>Case 4</td>
<td>Fuzzy Logic</td>
<td>4815</td>
<td>97.6</td>
</tr>
<tr>
<td>Case 4a</td>
<td>Fuzzy Logic</td>
<td>4837</td>
<td>98.0</td>
</tr>
<tr>
<td>Case 4b</td>
<td>Fuzzy Logic</td>
<td>4896</td>
<td>99.2</td>
</tr>
<tr>
<td>Case 4c</td>
<td>Fuzzy Logic</td>
<td>4890</td>
<td>99.1</td>
</tr>
</tbody>
</table>
The additional observations in Table 22 show large feeder speed changes over 5 minutes for Cases 3a, 4b and 4c, successful optimisation during the run-up for Cases 4b and 4c and some feeder stoppages appearing for the Fuzzy Logic Control cases. These observations will be discussed in sections 4.3.2 and 4.3.3.

4.3.2 PID Control Performance

Figure 18 shows a 2 hour trend of CV01 conveyor power and feeder FE01 speed for the PID control strategy simulations subject to metal detection disturbances. Tuning of the conveyor power controller was unchanged from that used for the size distribution disturbance cases. The tuning was again found to be satisfactory with power controlled to the setpoint for simulation cases 3 and 3a.
Figure 18 CV01 Power Control under PID Control, metal detection disturbances.

Figure 19 shows a 12 hour trend of BN02 level and feeder FE01 speed for the PID control strategy simulations subject to metal detection disturbances. From the trend of bin level it is apparent that level was maintained close to setpoint for all of the simulation cases.

When a metal detection event occurred, discharge from the bin stopped which caused a sudden increase in the bin level. Level was successfully returned to setpoint in each case with a relatively rapid recovery occurring for Case 3a with the highest gain tuning and recovery time increasing for cases 3 and 3b as the controller gain decreased. This observation is consistent with the Lambda tuning approach where increasing the value of Lambda increases the time to arrest a disturbance and gives a controller with a lower proportional gain.
From the trend of feeder speed the effect of controller gain may be seen. Case 3a achieved the fastest recovery of level following a metal detection event but this required the feeder speed to vary over a large range. As controller gain decreased with cases 3 and 3b, changes in feeder speed were less dramatic. These observations explain the summary data shown in Table 22.

![Figure 19 BN02 Level Control under PID Control, metal detection disturbances.](image)

4.3.3 Fuzzy Logic Control Performance

Figure 20 shows a 2 hour trend of CV01 conveyor power and feeder FE01 speed for the Fuzzy Logic control strategy simulations subject to metal detection disturbances.

The effects of the iterative approach to tuning the Fuzzy Logic control strategy are apparent by examining each simulation case in turn. Simulation Case 4 with the
initial tuning of the Fuzzy Logic controller was not successful in optimising the circuit as feeder speed did not reach the maximum limit of 80% even though conveyor power was well below the setpoint. The situation for Case 4a with more aggressive tuning for Rule I (negative error, negative error change) was essentially the same which shows that this tuning change was ineffective in the early stage of the simulation. Case 4b had more aggressive tuning applied to Rule F (negative error, zero error change) which was able to increase the feeder speed to the maximum limit. However the overall effect for Case 4b was that the control system was unable to successfully control conveyor power, with the feeders stopped a total of 10 times followed by restarting at 40% speed. Simulation Case 4c with the modified membership function for conveyor power was successful in achieving maximum feeder speed and controlling conveyor power to the setpoint.

Figure 20 CV01 Power Control under Fuzzy Logic Control, metal detection disturbances.
Figure 21 shows a 12 hour trend of BN02 level and feeder FE01 speed for the Fuzzy Logic control strategy simulations subject to feed size disturbances.

Figure 21 BN02 Level Control under Fuzzy Logic Control, metal detection disturbances.

The iterative approach to tuning from Case 4 through Case 4c achieved a more effective optimisation of the circuit, with each change causing a decrease in the time for the bin level to achieve setpoint. Once the bin level was close to setpoint the level was controlled reasonably well for each tuning case. However there were feeder stoppages due to a high bin level for Case 4a at 11.5 hours simulation time and for Case 4c at 1.25 and 4.5 hours simulation time.

While Cases 4b and 4c optimised the circuit during the initial run-up period, this came at the expense of large changes in feeder speed once the bin level was
established near setpoint. This is in contrast to cases 4 and 4a where the feeder speed may be seen to be generally within a 20% range.

4.3.4 Discussion

The metal detection disturbances caused a much larger impact to the crushing circuit than the feed size change disturbances and as a result the action of the control strategies was greater as shown by the changes in feeder speed.

Control of the conveyor power was shown to be good when under PID control, with power maintained close to the setpoint with no feeder stoppages due to high power.

The highest gain tuning for the bin level PID controller once again achieved the highest production over a 12 hour period, but the larger disturbances caused by metal detections showed that the highest gain also produced the largest changes in feeder speed once bin level control was established. Selecting final tuning for the bin level controller may require productivity and overall stability to be balanced against each other. If productivity from the circuit is the over-riding factor then high gain tuning would be selected. However with a metal detection event causing a relatively short duration level disturbance, the large change in speed could be viewed as unnecessary, or even considered to be increasing operating risk and a control approach that produces a more gradual change in speed may be favoured. Hence lower gain tuning may be preferred.

Fuzzy Logic Control was shown to be capable of achieving productivity that was less than but compared favourably to that achieved from the PID control strategy. However to achieve performance approaching that of PID control an iterative
approach to the tuning was shown to be necessary. This approach involved reviewing the results of a simulation, identifying opportunities for enhancing the performance, adjusting the tuning constants for the appropriate rule and re-running the simulation to obtain additional results. In order for this process to be effective it must be possible to determine the strength of each rule in the Fuzzy Logic Controller as a function of time so that tuning for the correct rule may be changed. There may be some risk that the overall tuning process could become excessively long, but for some applications the benefit that is achieved may outweigh the additional effort.

In addition to the tuning, performance of the Fuzzy Logic Controller was shown to be influenced by the structure of the controller, namely the membership functions for each rule and the output change assigned to each rule. Modifying the membership function for conveyor power error with no change in tuning parameters was successful in eliminating feeder stoppages due to high conveyor power. The membership function was changed to expand the range of zero error and to have the transition between negative error and zero error occur over a wider range, as shown in Figure 22. The original structure of the Fuzzy Logic Controller had 5 output changes assigned to the 9 Fuzzy Logic rules as shown in Table 13. During the iterative tuning of the Fuzzy Logic Controllers increased production was achieved when the output change for negative error zero error change was changed from Positive Small to Positive Large, as shown in Figure 23. As both the form of the membership functions and the rule outputs are set during the design process for the Fuzzy Logic Controller, this shows that the design choices made may need to be re-evaluated during the tuning process in order to fully optimise the control.
Reconsidering the design of the rule outputs for the Fuzzy Logic Controller can be taken a little further: instead of having output changes that are common to the Fuzzy Logic rules, modify the design to have 9 output changes with one assigned to
each rule. This concept is shown in Figure 24 where the labels A to I signify the independent rule output tunings. This would allow the output change of each rule to be optimised in isolation, eliminating the situation where, for example, modifying the zero change for the negative error increasing error change rule would also affect the controller actions for the zero error zero error change and positive error negative error change rules. With performance of the Fuzzy Logic Controller improved at each stage of the iterative tuning, this suggests that with additional tuning effort the performance could equal or exceed that of the PID Control Strategy. Independent tuning for each rule may be helpful in this regard.

**Figure 24 Fuzzy Logic Controller with Independent Output Changes.**
CHAPTER 5.  CONCLUSIONS

5.1 General

Evaluating the performance of the constraint control strategies using a dynamic simulation of a crushing circuit was found to be a valuable technique as the simulation ran faster than real time which allowed the effect of any changes to be assessed rapidly. This was of particular importance when tuning the Fuzzy Logic controller where it was necessary to evaluate the performance and determine which fuzzy logic rules were active so that the appropriate tuning change could be made.

5.2 Answers to Research Questions

RQ1: Can a dynamic model of a crushing circuit be built that includes circuit constraints?

The answer to this question is clearly yes. Crushing circuits are typically subject to internal limits such as bin level, drive power and screen loading that are set for equipment protection purposes or to maintain the efficiency of the process. If the internal limits are exceeded, production will be restricted and so these limits are known as constraints. This work extended the use of constraints within crushing circuit simulations by demonstrating the use of a dynamic model for belt conveyor power. The belt conveyor power model was developed from a standard calculation for belt conveyor power and so is expected to show good agreement with data from an operating conveyor. However the model was not validated due to the lack of access to plant data. Combining the new conveyor power constraint with the bin level constraint from earlier crushing circuit simulations has produced a simulation with the potential for an improved representation of a real world crushing circuit.
RQ2: What are the relative advantages of a PID Control and a Fuzzy Logic Control strategy when applied to the control of a crushing circuit?

The simulated crushing circuit was used to evaluate the performance of PID Control and Fuzzy Logic Control strategies used to control conveyor power and bin level. The two control strategies achieved a total production over a 12 hour period that was within 1% with the highest production obtained from the PID control strategy with the highest gain tuning. PID control appears to have the advantage in terms of production but the high gain tuning produced wide variation in the main feeder speed due to disturbances. While productivity is a key factor other factors such as overall circuit stability may be equally important.

The PID control strategy was found to have the advantage of straightforward tuning using established tuning rules which produced controllers that worked well the first time.

The Fuzzy Logic control strategy had the advantage of tuning that is somewhat flexible. With an iterative approach to tuning it was shown that it is possible to improve the productivity of the circuit and suppress other effects such as exceeding the conveyor power limit. Configuring a Fuzzy Logic Controller that had independent tuning for each Fuzzy Logic rule would provide additional flexibility with the potential for further optimisation and possibly achieving performance that is superior to a PID Control strategy.
RQ3: What is the relative level of effort required to bring a PID Control and a Fuzzy Logic Control strategy into operation?

The level of effort to implement each control strategy and achieve reasonable performance was assessed during the configuration and tuning process. It was found that the PID control strategy required less configuration effort to implement and tune compared to the Fuzzy Logic control strategy as less design input was required and controller tuning was straightforward.

Each control strategy required some decisions to be taken with regard to the design. PID controllers required the controller type (P, PI or PID) and form (Ideal or Parallel) to be selected. There were more design decisions involved in developing a Fuzzy Logic controller due to the need to specify membership functions, fuzzy logic rules and controller output actions for each rule.

To bring the control strategy into operation the controllers must be tuned and it was found that more effort was required to tune the Fuzzy Logic controller. Tuning the PID controllers was relatively straightforward using the Lambda tuning method which used a single tuning parameter and results of the process characterization to obtain the tuning parameters. In contrast it was necessary to adopt an iterative tuning approach for the Fuzzy Logic controllers as established tuning methods were not available. During the iterative tuning it was shown that some design decisions can limit the performance of the controller and so may need to be reconsidered in order to fully optimise performance.

5.3 Opportunities for Future Work

This masters’ thesis was focused on developing a dynamic model of a typical crushing circuit and using the dynamic model to evaluate the configuration and
performance of two advanced control strategies. During the modelling and simulation of the crushing circuit several ideas for future work were generated.

The dynamic model for belt conveyor power should be validated by comparing the predicted and actual power of several conveyors subject to a varying throughput. Validation would allow the model to be deployed confidently in situations where simulating an actual crushing circuit was required.

The dynamic simulation of a crushing circuit could be improved by including additional constraints such as jaw crusher level, jaw and cone crusher power and screen load. Including additional constraints would produce a dynamic simulation that better represents a real world crushing circuit and this would in turn provide scope to assess additional control strategies.

Fuzzy Logic controllers with independent tuning for each of the fuzzy logic rules should be examined in simulation studies. As highlighted in this work, independent tuning will allow each rule to be optimised without affecting the performance of other rules, an approach which may achieve a superior performance outcome.

Developing configuration and tuning guidelines for Fuzzy Logic controllers has the potential to reduce the time and effort required to bring the controller into operation. If this information was available the design and tuning of a Fuzzy Logic control system could be approached in an efficient manner similar to what is possible when established tuning rules are applied to PID controllers.
REFERENCES


[41] W. Cronje, private communication, Feb. 2018

A. CRUSHING CIRCUIT MODEL PARAMETERS

The tables that follow list the parameters that were used to configure the various unit models in the simulation.

Table 23 – Belt Conveyor Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CV01</th>
<th>CV02</th>
<th>CV03</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td>120</td>
<td>110</td>
<td>70</td>
</tr>
<tr>
<td>H (m)</td>
<td>30</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>S (m/s)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>F_e (-)</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>F_l (-)</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>t_f (m)</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Q (kg/m)</td>
<td>103</td>
<td>103</td>
<td>103</td>
</tr>
</tbody>
</table>

Table 24 – CV01 Conveyor Segments.

<table>
<thead>
<tr>
<th>Segment</th>
<th>L (m)</th>
<th>H (m)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>SC01 to CR01</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2.5</td>
<td>CR01 to CR02</td>
</tr>
<tr>
<td>3</td>
<td>106</td>
<td>26.5</td>
<td>CR02 to end</td>
</tr>
<tr>
<td>Total</td>
<td>120</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>
### Table 25 – Jaw Crusher CR01 Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Td</td>
<td>5</td>
<td>s</td>
<td>Deadtime</td>
</tr>
</tbody>
</table>

### Table 26 – Cone Crusher CR02 Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.7</td>
<td>t</td>
<td>Capacity</td>
</tr>
<tr>
<td>R&lt;sub&gt;max&lt;/sub&gt;</td>
<td>950</td>
<td>t/h</td>
<td>Maximum Rate</td>
</tr>
<tr>
<td>L&lt;sub&gt;choke&lt;/sub&gt;</td>
<td>60.0</td>
<td>%</td>
<td>Choke Level</td>
</tr>
<tr>
<td>K</td>
<td>7.675</td>
<td>1/%</td>
<td>Throughput</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calc. Parameter</td>
</tr>
</tbody>
</table>

### Table 27 – Bin BN01 Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>720</td>
<td>t</td>
<td>Capacity</td>
</tr>
<tr>
<td>n</td>
<td>100</td>
<td>-</td>
<td>layers</td>
</tr>
</tbody>
</table>
Table 28 – Feeder Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FE01</th>
<th>FE02</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>10</td>
<td>12</td>
<td>Gain (t/h per %)</td>
</tr>
<tr>
<td>T</td>
<td>5</td>
<td>5</td>
<td>Time const. (s)</td>
</tr>
<tr>
<td>T_d</td>
<td>5</td>
<td>5</td>
<td>Deadtime (s)</td>
</tr>
<tr>
<td>R_{max}</td>
<td>800</td>
<td>1100</td>
<td>Max rate (t/h)</td>
</tr>
</tbody>
</table>

Table 29 – Screen Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SC01</th>
<th>SC02</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_d</td>
<td>5</td>
<td>5</td>
<td>Deadtime (s)</td>
</tr>
<tr>
<td>OS</td>
<td>100</td>
<td>100</td>
<td>% to oversize</td>
</tr>
<tr>
<td>C1</td>
<td>100</td>
<td>100</td>
<td>% to oversize</td>
</tr>
<tr>
<td>C2</td>
<td>0</td>
<td>100</td>
<td>% to oversize</td>
</tr>
<tr>
<td>I1</td>
<td>0</td>
<td>100</td>
<td>% to oversize</td>
</tr>
<tr>
<td>I2</td>
<td>0</td>
<td>100</td>
<td>% to oversize</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>% to oversize</td>
</tr>
</tbody>
</table>
Table 30 – Protection Parameters.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Interlock</th>
<th>Action</th>
<th>Restart Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV01</td>
<td>Power &gt; 155</td>
<td>Stop FE01</td>
<td>Power &lt; 150</td>
</tr>
<tr>
<td>BN02</td>
<td>Level &gt; 85</td>
<td>Stop FE01</td>
<td>Level &lt; 80</td>
</tr>
<tr>
<td>BN02</td>
<td>Level &lt; 10</td>
<td>Stop FE02</td>
<td>Level &gt; 30</td>
</tr>
<tr>
<td>CR02</td>
<td>Level &gt; 90</td>
<td>Stop FE02</td>
<td>Level &lt; 85</td>
</tr>
</tbody>
</table>
B. DYNAMIC MODEL FOR BELT CONVEYOR POWER

The following equation for calculating belt conveyor power was provided by a conveyor manufacturer [18], comprising three terms: empty (unloaded) power, loaded power and power to raise the conveyor load:

\[
P (kW) = \frac{F_e(L + t_f)3.6QS}{367} + \frac{F_l(L + t_f)C}{367} + \frac{CH}{367}
\]  \hspace{1cm} (B1)

where:

- \( F_e \) and \( F_l \) are the empty and loaded friction factors respectively
- \( L \) is the conveyor centre to centre length (m)
- \( t_f \) is the terminal friction constant expressed as an equivalent length (m)
- \( Q \) is the mass of rotating parts (kg/m)
- \( S \) is the conveyor speed (m/s)
- \( C \) is the conveyor capacity (t/h)
- \( H \) is the elevation change (m)
- 367 is the approximate value of \( \frac{3600}{g} \)
- \( g \) is the acceleration due to gravity (m/s\(^2\))

As all terms in the equation are constants for a given conveyor the calculation is static and is typically used to determine the power required for a fully loaded belt.
The derivation that follows shows how the above equation may be modified to provide a dynamic power estimate. The derivation is in two parts:

- Power required to raise the conveyor load.
- Power required to overcome friction effects.

**Part 1: Power to raise conveyor load.**

Figure 25 shows a belt conveyor of length $L$ lifting material through a height $H$ operating at a speed $u$, with the length divided into $n$ segments of length $l$. Consider segment $i$ with a mass $m_i$ in kg/m.

![Belt Conveyor with n segments lifting material.](image)

**Figure 25 Belt Conveyor with $n$ segments lifting material.**

From a force balance on element $i$, the downward force due to gravity is equal to the upward force from the conveyor:

$$F_c = m_i l g$$  \hspace{1cm} (B2)  

In time $\Delta t$, material on the conveyor will be lifted through a height $\Delta h$. From the conveyor length and overall height change we obtain:
\[
\Delta h = \frac{H}{L} u \Delta t
\]  

(B3)

Work is given by force multiplied by displacement. In time \( \Delta t \), the work done by the conveyor on element \( i \) is:

\[
W_i = F_c \Delta h
\]  

(B4)

\[
W_i = m_i l g \frac{H}{L} u \Delta t
\]  

(B5)

Total work done by the conveyor is the sum of \( W_i \) for all length elements:

\[
W = \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} m_i l g \frac{H}{L} u \Delta t
\]  

(B6)

Conveyor power is the rate of work:

\[
P = \frac{W}{\Delta t} = \sum_{i=1}^{n} m_i l g \frac{H}{L} u
\]  

(B7)

Now, if the conveyor is uniformly loaded, \( m_i = m \) kg/m for all elements. The summation in the above expression is:

\[
\sum_{i=1}^{n} m_i l = n m l
\]  

(B8)

With \( L = n l \), the expression for conveyor power simplifies to:

\[
P = m g H u
\]  

(B9)
Given a uniform loading in kg/m and uniform conveyor speed in m/s, the conveyor capacity in tonnes per hour is given by:

\[
C = m \, u \, \frac{3600}{1000} = m \, u \, 3.6 \tag{B10}
\]

Rearranging:

\[
m \, u = \frac{C}{3.6} \tag{B11}
\]

and substituting into the power expression gives:

\[
P (W) = \frac{C}{3.6} \, g \, h \tag{B12}
\]

\[
P (kW) = C \, h \, \frac{g}{3600} \tag{B13}
\]

Which is the third term in equation (B1).

Now consider the situation when the conveyor is not uniformly loaded. The summation term gives the total mass on the conveyor:

\[
\sum_{i=1}^{n} m_i \, l = m_T \tag{B14}
\]

and the expression for power becomes:

\[
P = m_T \, g \, \frac{H}{L} \, u \tag{B15}
\]
This expression shows that if the total mass on the conveyor can be
determined with time, the conveyor power due to lifting the material may be
calculated with time.

**Part 2: Power due to friction effects.**

Consider a belt conveyor of length $L$ operating at a speed $u$, with the length
divided into $n$ segments of length $l$. Consider segment $i$ with a mass $m_i$ in kg/m that is
experiencing a friction force $F_F$ as shown in Figure 26.

![Figure 26 Belt Conveyor segment $i$ experiencing friction.](image)

Neglecting the change in elevation the friction force on element $i$ is
proportional to the gravitational force:

$$F_F = f F_G = f m_i l g$$  \hspace{1cm} (B16)

where the proportionality constant $f$ is the friction factor.

In time $\Delta t$, element $i$ moves through a horizontal distance $\Delta l$ which is given
by:

$$\Delta l = u \Delta t$$  \hspace{1cm} (B17)

Work is given by force multiplied by displacement. In time $\Delta t$, the work done
by the conveyor on element $i$ is:
\[ W_i = F_F \Delta l \quad \text{(B18)} \]

\[ W_i = f m_i l g u \Delta t \quad \text{(B19)} \]

Total work done by the conveyor is the sum of \( W_i \) for all length elements:

\[ W = \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} f m_i l g u \Delta t \quad \text{(B20)} \]

Conveyor power is the rate of work:

\[ P = \frac{W}{\Delta t} = \sum_{i=1}^{n} f m_i l g u \quad \text{(B21)} \]

As before, considering the case when the conveyor is uniformly loaded, \( m_i = m \) kg/m for all elements. The summation in the above expression is:

\[ \sum_{i=1}^{n} m_i l = n m l \quad \text{(B22)} \]

With \( L = n l \), the expression for conveyor power simplifies to:

\[ P = f m L g u \quad \text{(B23)} \]

As performed in Part 1, substituting:

\[ m u = \frac{C}{3.6} \quad \text{(B24)} \]

into the power expression gives:
\[ P(W) = fLg \frac{C}{3.6} \quad \text{(B25)} \]

\[ P(kW) = fLg \frac{C}{3.600} \quad \text{(B26)} \]

Which is in the same form as the second term in equation (B1).

Now consider the situation when the conveyor is not uniformly loaded. The summation term gives the total mass on the conveyor:

\[ \sum_{i=1}^{n} m_i l = m_T \quad \text{(B27)} \]

and the expression for power becomes:

\[ P = f m_T g u \quad \text{(B28)} \]

This expression shows that if the total mass on the conveyor can be determined with time, the conveyor power due to friction may be calculated with time.

**Summary**

The above derivation has examined the capacity dependent terms in equation B1 and has shown that the static power calculation may be converted to a dynamic power calculation if the total mass of material on the conveyor can be determined with time.
C. STEADY STATE MASS BALANCE

A steady state mass balance was configured in Microsoft Excel and was used to determine the following parameters:

- Nominal feed size distribution for the crushing circuit, obtained by varying the 50% passing size to obtain 30% fines in the combined product from the Vibrating Grizzly Screen and the Jaw Crusher.

- Selection function for the Cone Crusher, which was varied to obtain a recirculating ratio for the Cone Crusher equal to 1.9.

This section describes the approach that was used to obtain the parameter values from the mass balance.

Nominal Feed Size Distribution

The nominal feed size distribution was obtained by solving the mass balance for the Feeder, Vibrating Grizzly Screen and Jaw Crusher as follows:

1. Assign Screen Classification Function.
2. Assign Product Size Distribution from Jaw Crusher
3. Set feed rate to the circuit.
4. Guess $x_{50}$ for the size distribution based on the Swebrec function.
5. Calculate the size distribution.
6. Solve the mass balance to obtain the combined product from the screen and crusher.
7. Calculate the Fines fraction in the combined product.
8. If the Fines fraction was not equal to 30%, return to Step 4.
9. If the Fines fraction was equal to 30%, end.

**Cone Crusher Selection Function**

The Cone Crusher Selection Function was obtained by solving the mass balance for the entire circuit as follows:

1. Assign Screen Classification Functions, Jaw Crusher Product Size Distribution and use the Nominal Size Distribution.
2. Set Feed Rate to the circuit.
3. Guess Selection Function for the Cone Crusher.
4. Solve the mass balance iteratively.
5. Calculate the Recirculating Ratio for the Cone Crusher.
6. If the Recirculating Ratio was not equal to 1.9, return to Step 3.
7. If the Recirculating Ratio was equal to 1.9, end.

The above approach required an iterative approach to obtain a steady state solution for the mass balance due to the recycling of oversize material through the Cone Crusher.
D. CONTROLLER TUNING AND CONFIGURATION

This section describes how the process were characterised and how this information was used to determine the tuning for the PID and Fuzzy Logic Controllers. Configuration of the Fuzzy Logic Controllers is also included.

Process Characterisation – CV01 Power

The response of CV01 Power was determined by performing a step change to feeder FE01 Speed. Figure 27 shows the response of CV01 power to a 10% change in feeder speed. Table 31 shows the values that were obtained from the step test.

Figure 27 Conveyor CV01 Power Process Reaction Curve.
### Table 31 – Step Test Data for CV01 Power.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Power</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>53.65 kW</td>
<td>50 %</td>
</tr>
<tr>
<td>Final</td>
<td>92.93 kW</td>
<td>60 %</td>
</tr>
<tr>
<td>Change</td>
<td>9.29 kW</td>
<td>10 %</td>
</tr>
<tr>
<td>Range</td>
<td>0-180 kW</td>
<td>0-100 %</td>
</tr>
<tr>
<td>Change (fraction)</td>
<td>0.052</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Process Gain is the change in the process value (power) divided by the change in manipulated value (speed) with changes expressed as a fraction of the full scale value. Process Gain $K_P$ is then equal to 0.516.

Process Time Constant and Deadtime were found using the method described by Wade [38], where the time for 10% and 90% of the change in process value are found. From the step test, $t_{10}$ and $t_{90}$ were found to be 19 and 68 seconds respectively.

Process Time Constant is given by:

$$
\tau_P = 0.45(t_{90} - t_{10}) \tag{D1}
$$

Process Deadtime is given by:

$$
\tau_D = t_{10} - 0.1\tau_P \tag{D2}
$$

Evaluating the above expressions gave 22 seconds for the Process Time Constant and 17 seconds for the Process Deadtime.
Process Characterisation – BN02 Level

The response of BN02 Level was determined by performing a step change to Feeder FE01 Speed. Figure 28 shows the response of CV01 power to a 10% change in feeder speed and Table 32 shows the values that were obtained from the step test.

![Figure 28 Bin BN02 Power Process Reaction Curve.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Level</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.0 %/h</td>
<td>50 %</td>
</tr>
<tr>
<td>Final</td>
<td>13.9 %/h</td>
<td>60 %</td>
</tr>
<tr>
<td>Change</td>
<td>13.9 %/h</td>
<td>10 %</td>
</tr>
<tr>
<td>Range</td>
<td>0-100 %</td>
<td>0-100 %</td>
</tr>
<tr>
<td>Change (fraction)</td>
<td>0.139 per hour</td>
<td>0.1</td>
</tr>
</tbody>
</table>
For an integrating process, the Process Gain is the rate of change produced in the process value (level) divided by the change in manipulated value (speed) with changes expressed as a fraction of the full scale value. Process Gain $K_P$ is then equal to 1.389 which has units % level per hour per % speed.

**PID Controller Tuning – CV01 Power**

The Lambda Tuning Method [37] was used to tune CV01 Power Controller. For a first order process with process gain $K_P$, process time constant $\tau_P$, and process deadtime $\tau_D$ known, PID controller gain and time constant are determined as follows [37]:

$$K_C = \frac{1}{K_P \lambda + \tau_D} \quad \text{(D3)}$$

$$\tau_C = \tau_P \quad \text{(D4)}$$

Where the tuning parameter $\lambda$ is the desired response time constant for the first order process. For CV01 Power Controller, $\lambda$ was set equal to $\tau_P$ to obtain an aggressively tuned controller with the aim of avoiding feeder stoppages due to excessive conveyor power draw.
**PID Controller Tuning – BN02 Level**

The Lambda Tuning Method [37] was used to tune BN02 Level Controller. For an integrating process with process gain $K_P$ and process deadtime $\tau_D$ known, PID controller gain and time constant are determined as follows [37]:

$$K_C = \frac{1}{K_P} \frac{(2\lambda + \tau_D)}{(\lambda + \tau_D)^2}$$

(D5)

$$\tau_C = 2\lambda + \tau_D$$

(D6)

Where the tuning parameter $\lambda$ is the desired time to arrest a change in throughput.

**Fuzzy Logic Controller Tuning and Configuration – CV01 Power**

The output change from the Fuzzy Logic Controller is shown in equation (D7) where $z$ is the output change, $x$ is the error, $y$ is the error change and $A$, $B$ and $C$ are tuning parameters.

$$z = Ax + By + C$$

(D7)

Tuning parameters for CV01 Power Fuzzy Logic Controller were obtained as follows:

The process gain for CV01 Power expressed in raw form is 0.929 kW/%. The reciprocal of this value is 1.076 %/kW, which is the change in feeder speed required to produce a power change of 1 kW.
To find the value of $C$ in equation (D7), consider the case where power is steady with an error of 2 kW. From the process gain, a 2 kW change in power is equivalent to a feeder speed change of $2.153\%$, so the controller must change the feeder speed by $2.153\%$ to bring power to the setpoint. If the speed change is achieved in twice the residence time on CV01, equation (D7) can be written with units of % per second:

$$\frac{2.153}{120} = Ax + By + C \quad (D8)$$

With $A=0$ for all tunings and $B=0$ in this case, the value of $C$ is found to be equal to $0.018\%$ speed per second.

To find the value of $B$ in equation (D7), consider the case where power is at setpoint but is increasing at a rate of 1 kW/s. From the process gain, 1 kW is equivalent to $1.076\%$ speed, and so the controller must change the feeder speed by $1.076\%$ to bring the rate of change to zero. Again, rewriting equation (D7) with units of % per second:

$$1.076 = Ax + By + C \quad (D9)$$

With $A=0$ for all tunings, and setting $C=0$, the value of $B$ is found to be equal to $1.076\%$ speed per kW per second.

When obtaining the tuning parameters for CV01 Power Fuzzy Logic Controller, an error of 2 kW and an error change of 1 kW/s were used. These values were used in the corresponding membership functions to set the point of transition between the zero condition and the positive or negative conditions, as shown in Figure
During the simulations it became necessary to try and suppress excessive power on CV01 which was causing the feeder to stop. In order to achieve this the membership function for the error was changed to have a more gradual transition between *negative* and *zero* as shown in Figure 30.

**Figure 29 CV01 Power Fuzzy Logic Controller Membership Functions - Original.**

**Figure 30 CV01 Power Fuzzy Logic Controller – Modified Error Membership Function.**
Fuzzy Logic Controller Tuning and Configuration – BN02 Level

Adopting a similar approach to that used for CV01 Power, tuning parameters for BN02 Level Fuzzy Logic Controller were obtained as follows:

The process gain for BN02 Level expressed in raw form is 1.389 % level per hour per % feeder speed. The reciprocal of this value is 0.72 % speed per % level per hour, which is the change in feeder speed required to produce a change in level of 1 % per hour.

To find the value of $C$ in equation (D7), consider the case where level is steady with an error of 5 %. From the process gain, a 5 % change in level is equivalent to a 3.6 % change in feeder speed, and so the controller must change the feeder speed by 3.6 % to return level to the setpoint. If the controller is to return level to setpoint in one hour then equation (D7) can be written with units of % per second:

$$\frac{3.6}{3600} = Ax + By + C$$  \hspace{1cm} (D10)

With A=0 for all tunings and B=0 in this case, the value of $C$ is found to be equal to 0.001 % speed per second.

To find the value of B in equation (D7), consider the case where level is at setpoint but is increasing at a rate of 6 % per hour. From the process gain, a level change of 6 % per hour is equivalent to a speed change of 4.32 % per hour, and so the controller must change the feeder speed by 4.32 % to bring the rate of change to zero. Rewriting equation (D7) with units of % per second:
\[
\frac{4.32}{3600} = Ax + By + C
\]  \hspace{1cm} (D11)

With \(A=0\) for all tunings, and setting \(C=0\), the value of \(B\) may be found. If the rate of level change is determined over 1 minute, 6 % level per hour is equivalent to 0.1 % level per minute, and so the value of \(B\) is equal to 0.012 % speed per % per minute.

When obtaining the tuning parameters for BN02 Level Fuzzy Logic Controller, and error of 5 % and an error change of 6 \%/h (=0.1 \%/min) were used. These values were used in the corresponding membership functions to set the point of transition between the zero condition and the positive or negative conditions, as shown in Figure 31.

![Figure 31 BN02 Level Fuzzy Logic Controller Membership Functions.](image-url)